The Town Effect:

Dynamic Interaction between a Group of Structures and Waves in the Ground

by

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Abstract

In a conventional approach, the mechanical behaviour of a structure subjected to seismic or blast waves is treated separately from its surroundings, and in many cases, the dynamic coupling effect between multiple structures and the waves propagating in the ground is disregarded. However, if many structures are built densely in a developed urban area, this dynamic interaction may not become negligible. The first purpose of this contribution is to briefly show the effect of multiple interactions between waves and surface buildings in a town. The analysis is based on a recently developed, fully-coupled rigorous mathematical study, and for simplicity, each building in the town is represented by a rigid foundation, a mass at the top and an elastic spring that connects the foundation and mass. The buildings stand at regular spatial intervals on a linear elastic half-space and are subjected to two-dimensional anti-plane vibrations. It is found that the buildings in this model significantly interact with each other through the elastic ground and the resonant (eigen) frequencies of the collective system (buildings or town) become lower than that of a single building with the same rigid foundation. This phenomenon may be called the "town effect" or "city effect." Then, secondly, it is shown that the actual, unique structural damage pattern caused by the 1976 Friuli, Italy, earthquake may better be explained by this "town effect," rather than by investigating the seismic performance of each damaged building individually. The results suggest that it may also be possible to evaluate the physical characteristics of incident seismic/blast waves "inversely" from the damage patterns induced to structures by the waves.

Keywords Collective behaviour, Earthquake hazard, City effect, Town effect, Dynamic interaction.

Abbreviations: Uenishi, Town Effect

1 Introduction

The devastating 2009 L'Aquila earthquake in the mountainous region of central Italy has renewed our appreciation of earthquake engineering in light of rock mechanics and dynamics (see e.g., INGV QUEST 2009, Uenishi 2009). Conventional analyses in engineering seismology, however, usually handle the mechanical behaviour of each structure independently, and the interaction between structural vibrations and the waves in the ground (rock mass, soil) is most often neglected – Although structures, either on the surface or in the ground, do exist next to each other in a more developed environment, namely, in a town or a city like L'Aquila. Instead, the structure itself is assumed to consist of more complex and realistic components and the vibration characteristics are analysed in great detail, but at present, it is not so certain that these conventional methods are valid for analysing the seismic performance of a group of structures densely built in an urban area. It may be difficult to conclude that the dynamic interaction between multiple structures and the waves propagating in the ground (structure-wave-structure interaction) is insignificant.

Figure 1

Indeed, one of the historically largest earthquakes in Italy, the 1976 quake in the Friuli region, may have possibly posed a question regarding the (non)existence of such dynamic coupling structure-wave-structure interaction: A photograph taken in the epicentral region in 1976 (Fig. 1) shows a surprisingly "regular" (periodic) damage distribution where each adjacent building has experienced completely different mechanical behaviour – One building totally collapsed while the next one was almost undamaged, and this alternate "collapsed-undamaged" pattern is repeated further. It might be easier to explain that the "dissimilar but regular" damage distribution is attributed to, say, the strength or construction year of each building. However, if the buildings with short separation distances are subjected to almost the same (frequency components of) seismic waves under very similar geological situations, the structural damage may be, to some extent, also comparable. Figure 1 suggests, even when we accept the importance of the causes like the fragility of each individual structural component, it is not simple to explain, systematically and comprehensively from these "plausible" causes, the generation of the clearly alternate damage levels in such a short distance. At least, it is worthwhile to try to mechanically describe the phenomenon observed in Fig. 1 in a more straightforward way, with possible dynamic structure-wave-structure interaction taken into consideration.

Although certain research was conducted at an earlier stage, for example, on anti-plane vibration of several shear walls on an elastic half-space (Wong and Trifunac 1975), more interest in the interaction between multiple structures and the ground appeared after the 1985 Michoacan earthquake that had generated severe damage to Mexico City – nine years after the Friuli earthquake. The difficulties of classical computational methods in matching the seismic records have given the idea that part of the seismic energy transmitted to the buildings may be re-transferred back into their neighbourhood through multiple interactions between vibrating structures and the waves in ground. Recent simulations based on different models representing a city (or town) and various numerical techniques (e.g., Green functions, finite elements) seem to support this idea (Ghergu and Ionescu 2009): The studies regarding the effect of building vibration on ground motion have shown that the vibrations of multiple structures may radiate waves into the ground through their foundations (Wirgin and Bard 1996, Clouteau and Aubry 2001, Guéguen et al. 2002, Tsogka and Wirgin 2003, Boutin and Roussillon 2004, Kham et al. 2006). It has been reported, for example, that during the Michoacan earthquake, some anomalously long and strong ground motions due to the interaction between urban structures caused considerable damage to the developed area. In Mexico City, the mechanical characteristics of the alluvial layers and the buildings seem favourable for structure-ground coupling and, together with the urban (built) environment of the city, they might have resulted in that significant city-scale vibration effect (Kham et al. 2006). The investigations performed after that earthquake using several different models of a city and numerical techniques include: A two-dimensional anti-plane study to describe the diffraction pattern of waves in the surface layer due to the influence of a periodic assembly of blocks (buildings) (Wirgin and Bard 1996); Analysis of an idealised two-dimensional city that consists of ten non equally-spaced, non equally-sized, homogenised blocks anchored in a soft soil layer overlying a hard rock (half-space) and displays strong seismic response inside the blocks (Tsogka and Wirgin 2003); and three-dimensional computations utilising boundary element method (Clouteau and Aubry 2001). Other research addresses the structure-ground interaction theoretically (Kham et al. 2006): The large-scale effect of a city is estimated by summing up the contribution from each building represented by a single oscillator (Guéguen et al. 2002); and the multiple interactions between periodically-located simple oscillators are discussed from a "macroscopic" city-scale point of view (Boutin and Roussillon 2004). However, precise, more "microscopic" vibration behaviour of each building in a city has not been thoroughly identified yet.

In the following, first, by utilising a simplified model of a town and the mathematical technique originally developed in (Ghergu and Ionescu 2009), a fully-coupled elastodynamic analysis will be performed to clarify the mechanical effect of multiple interactions between waves and surface buildings in a town. It will be shown that, due to the dynamic interaction through (the waves in) the ground, the eigenfrequencies of the collective multiple-building system become lower than the resonant frequency of a single building. This shift of eigenfrequencies may be called the "town effect" (or "city effect"). Then, the generation mechanism of the alternate structural damage levels caused by the 1976 Friuli earthquake (Fig. 1) will be investigated and it will be shown that the damage pattern may have been actually induced by the "town effect."

2 Analytical Basis

Consider a two-dimensional anti-plane problem of a homogeneous, isotropic linear elastic half-space, representing rock mass or soil near the free surface (ground). It is supposed, for simplicity, that N buildings are uniformly distributed in a town (total length $2l_t$) located on the surface ($y = 0$) along the x-axis at $-l_t < x < l_t$, with $2l_b$ being the width of the rigid foundation of each building located at $a_j < x < b_j$ ($= a_j + 2l_b$) and d being equal separation distance ($1 \leq j \leq N$; Fig. 2a). The foundation itself has no height. The anti-plane horizontal displacement in the z-direction, $w(x, y, t)$, satisfies the equation of motion in the half-space

Figure 2

$$\rho \frac{\partial^2 w}{\partial t^2} = \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right), \tag{1}$$

where ρ and μ are the mass density and shear modulus of the ground, respectively. The rigid and stress-free boundary conditions along the x-axis on $y = 0$ are given by

$$w(x, 0, t) = w^{m0}{}_j(t), \qquad \text{(for rigid foundations; } a_j < x < b_j,\ 1 \le j \le N)$$

$$\mu \frac{\partial w}{\partial y}(x,0,t) = 0\,, \qquad \text{(elsewhere)} \tag{2}$$

respectively, with $w^{m0}{}_j(t)$ being the anti-plane horizontal displacement of the foundation of the j-th building in the town. Further, assume that each building has the same mechanical characteristics and consists of a foundation (mass per unit length m_0), a mass m_1 (again, per unit length) at the top and the elastic spring connecting the foundation m_0 and mass m_1. The elastic spring produces resistant force that is proportional to the elastic modulus k and the relative anti-plane horizontal displacement of the mass m_1 with respect to the foundation m_0, i.e.,

$$m_1 \frac{d^2 w_j^{m1}(t)}{dt^2} = -k[w_j^{m1}(t) - w_j^{m0}(t)]\,,$$

$$m_0 \frac{d^2 w_j^{m0}(t)}{dt^2} = \int_{a_j}^{b_j} \mu \frac{\partial w(s,0,t)}{\partial y} ds + k[w_j^{m1}(t) - w_j^{m0}(t)]\,. \qquad (1 \le j \le N) \tag{3}$$

Here, $w^{m1}{}_j(t)$ is the anti-plane horizontal displacement of the mass at the top of the j-th building, and the relations $k = 2\mu_b l_b/h$ and $m_1 = 2\rho_b l_b h$ hold, with ρ_b, μ_b and h being the mass density, shear modulus and height of the building. Due to the dynamic interaction mathematically expressed by Eqs (1)-(3), the displacement amplitude of every foundation (or mass at the top) may become different from each other even for a single vibration frequency of the town (Fig. 2b).

In fact, a rigorous eigenvalue analysis briefly summarised in Appendix indicates such behaviour of buildings in the town. The eigenfrequencies of the vibrations of the town consisting of N identical buildings may be obtained by analysing the eigenvalues and eigenvectors of the $N \times N$ matrix T shown in Eq. (7) in Appendix (and then solving Eq. (6)). At this point, it may be convenient to introduce the normalised frequency ξ for the following discussion

$$\xi = \omega l_b / c_S , \tag{4}$$

where ω is the angular frequency of vibration and equal to $2\pi f$ (f: frequency), and c_S ($=\sqrt{\mu / \rho}$) is the shear wave speed of the linear elastic ground.

Figure 3

In Fig. 3, the normalised eigenfrequencies of all vibration modes are shown for a town of seven buildings having the identical mechanical characteristics (N = 7). In generating this figure (and also Fig. 4 below), it is assumed that d/l_b = 0.4, h/l_b = 2, m_1/m_0 = 1.5, ρ_b/ρ = 0.1 and $(c_S)_b/c_S$ = 1.5, as suggested by (Ghergu and Ionescu 2009) for European towns, with $(c_S)_b$ ($=\sqrt{\mu_b / \rho_b}$) being the shear wave speed in the buildings. With these geometrical and mechanical properties, the normalised eigenfrequencies for the first (k = 1), fourth (k = 4) and seventh (k = 7) vibration modes of the collective behaviour of the buildings (town) are found to be $\xi_k \equiv 2\pi f_k\, l_b/c_S$ = 1.071, 0.839 and 0.780, respectively. As seen in Fig. 3, the eigenfrequency ξ_k of each vibration mode [$1 \leq k \leq N$ (= 7)] is smaller than that of a single building with the same foundation ξ_0 (= 1.186), which shows the theoretical existence of the "town effect." Note also in Fig. 3 that the normalised eigenfrequency ξ_k lies in the range between the two eigenfrequencies ξ_0 [for a single m_1-k-m_0 (mass-spring-foundation) system] and ξ_∞ [= 0.75; a single (or an N-) m_1-k (mass-spring) system on a rigid half-space], as expressed by Eq. (9) in Appendix.

Figure 4

Figure 4 illustrates the associated eigenvector or distribution of the normalised displacement amplitudes of the foundations for the k-th vibration mode of the town: α^k_j (indicated by black rectangles) and $|\alpha^k_j|$ (shown by white rectangles where α^k_j is negative) for the identical town (N = 7, with j being the building number, $1 \leq j \leq N$ and $1 \leq k \leq N$, see Appendix). Note that, if normalised, the distribution of the displacement amplitudes of the masses m_1 at the top for the k-th mode is exactly the same as that for the foundations m_0 [refer to the expressions for $w^{m0}{}_j(t)$ and $w^{m1}{}_j(t)$ in Appendix]. Moreover, in this harmonic analysis, the normalised distributions of displacements are equivalent to those of velocities and accelerations. Therefore, if each building can be assumed to have (more or less) the same mechanical properties and the induced structural damage level is proportional to the (maximum of the absolute value of) displacement, velocity or acceleration experienced (at the foundation or the mass at the top), these diagrams can be compared to the ones showing the damage levels of buildings or photographs like Fig. 1. Figure 4a pertains to the first vibration mode (k = 1) where all displacements are in phase and the building in the middle of the town, number 4, may be subjected to the severest vibration and therefore more damage is expected to this building than to the others. This distribution is very similar to that for the seventh mode (k = 7, Fig. 4g), but only every second building moves in phase and the vibration is more "out-of-phase" in Fig. 4g. In the second mode (k = 2, Fig. 4b) the same building 4 in the middle experiences no dynamic impact, and like in the sixth mode (k = 6, Fig. 4f), specific buildings (2 and 6) are subjected to stronger vibrations. The third mode (k = 3, Fig. 4c) shows again the displacement of the building 4 is the largest one and also in the similar fifth mode (k = 5) every third building may have a larger displacement. Figures 4d indicates that every second building (j = 1, 3, 5, 7) is under much stronger vibration in the fourth mode (k = 4). Thus, Fig. 4, together with Fig. 3, clearly demonstrates that slight change in vibration frequencies can induce totally different dynamic behaviour of the town, which may not be systematically, or in a unified way, explained through conventional analyses handling each individual building separately.

Figures 3 and 4 show a "peculiar" feature in the sense that a smaller eigenfrequency seems to correspond to a "higher" (more "out-of-phase") mode of vibration, but as stated in detail in Discussion, this "reverse" order of eigenvectors may become also "normal" (i.e., a higher eigenfrequency corresponds to a "higher" vibration mode of the town), depending on the combination of geometrical and mechanical properties actually employed in the analysis.

3. Possible Example of the Seismic "Town Effect"

3.1. The 1976 Friuli, Italy, Earthquake

Here, based on the analytical results summarised above, the generation mechanism of seismic damage pattern observed in the Friuli region in 1976 (Fig. 1) will be studied.

The main shock (Richter magnitude M_L = 6.5) occurred on 6 May 1976, with the epicentre located about 25 km north of the city of Udine and the focal depth being some 10 km. The region of Friuli is located in the eastern sector of the Southern Alps where the orogenesis is related to the convergence of European and Adriatic plates. The main shock was preceded by an M_L = 4.5 foreshock, and followed by a large number of aftershocks, with the largest ones on 15 September 1976 (M_L = 6.1 and 5.8) (Cipar 1980, Zollo et al. 1997). The spectral response estimated from the main shock and aftershocks of this Friuli 1976-1977 earthquake sequence for the TLM1 (Tolmezzo-Ambiesta dam) accelerograph site at the top of a calcareous hill shows the dominant (peak) frequencies f_d of observed seismic waves near the epicentre to be about 2, 3.8, and 6-8 Hz. The estimation is based on four different methods: spectral-ratio-to-reference-site; generalised inversion; median response spectra predicted for a rock soil by European attenuation relations; and receiver-function technique (see Barnaba et al. 2007).

Figure 1 suggests, if we can assume the number of buildings in the affected "town" was seven and, again, if all buildings there had (approximately) the same mechanical properties and the damage level is proportional to the maximum acceleration (velocity or displacement) of each building, the town might have been collectively under the fourth vibration mode during the earthquake (compare Fig. 1 with 4d: Every second building may totally collapse under much stronger vibrations). If the observed values, the shear wave speed of the ground c_S = 225 m/s [Based on the c_S vertical profile defined by a shallow seismic refraction survey at TLM1 in 1977 (Barnaba et al. 2007)] and the length of each building $2l_b$ = 16 m (height h = 8 m), as well as the same geometrical and mechanical properties as in the last chapter, are employed, then, based on the analytical results, the original eigenfrequency of a single building with a rigid foundation may be evaluated approximately as $f_0 = \xi_0 c_S/(2\pi l_b) = 5.3$ Hz. This resonant frequency, in the typical natural frequency range of short reinforced concrete buildings, may be too high compared with the seismologically estimated dominant frequencies f_d = 2 and 3.8 Hz (and lower than the other f_d = 6-8 Hz), and it does not seem straightforward to explain the generation of damage pattern in Fig. 1 using this "conventional" resonant frequency for a single building. However, if the buildings in the town are treated collectively and the normalised eigenfrequencies ξ_k associated with the "town effect" are used, the dimensional eigenfrequencies $f_k = \xi_k c_S/(2\pi l_b)$ become approximately 4.8 Hz (first mode, $k = 1$), 3.8 Hz (fourth mode, $k = 4$), or 3.5 Hz (seventh mode, $k = 7$), respectively. These resonant frequencies, possibly causing serious damage only to certain buildings in the town, are lower than the original eigenfrequency of a single building f_0. One of the dominant frequencies f_d evaluated from the observations (3.8 Hz) lies in this range of "collective" eigenfrequencies f_k, and it is well comparable to that of the fourth vibration mode f_4, as expected from Figs. 1 and 4d. The other dominant frequencies (2 and 6-8 Hz) are either too low or too high for the resonance of the town or a single building. As stated earlier, slight difference in dominant wave frequency component gives totally dissimilar damage patterns, especially in the middle section of the town, and there are still many unknown or unconsidered factors in the model, but the present study may have shown one possible real example of the "town effect" and it also suggests that the physical characteristics of incident seismic (or blast) waves may be "inversely" evaluated from the structural damage patterns induced by the waves.

3.2. Discussion

The dynamic structure-wave-structure interaction has been studied and its crucial effect on the collective mechanical behaviour of a group of surface buildings (town) has been addressed, but as stated above, in Figs. 3 and 4 a smaller eigenfrequency is corresponding to a "higher" mode of vibration. Preliminary study on the effect of building height (h/l_b) and separation distance between each foundation (d/l_b) shows, besides clearly demonstrating the existence of the "town effect" again (e.g., for larger h/l_b), the order of eigenvectors may become also "normal": For example, if the same geometrical and mechanical properties except for longer separation distance $d/l_b = 2$ (i.e., the separation distance d and the building width $2l_b$ are the same) are used, a higher eigenfrequency is associated with a "higher" vibration mode of the town. This "reverse" or "normal" order of vibration modes (eigenvectors) is dependent on the combination of geometrical / mechanical properties used in the computations and it may possibly be attributed to the "fluctuating" nature of the Hankel and Bessel functions of the first kind of order zero, i.e., (real and imaginary parts of) $H_0^{(1)}(x)$ and $J_0(x)$, which appear in the analysis and take both positive and negative values depending on x: Similar "fluctuating" behaviour due to the introduction of Hankel and Bessel functions may be observed in the wave interaction problems in general, e.g., in the classical analysis of dynamic stress concentrations around a circular cavity in an infinitely extended, thin elastic plate during passage of plane longitudinal waves (Pao 1962) and also in the study of the seismic performance of dual tunnels, typically two tunnels running in parallel, at various depths (Uenishi and Sakurai 2008). The important and consistent points here are that, regardless of the order of eigenvectors, the distributions of the (absolute values of) displacement amplitudes, such as shown by the black and white rectangles for $|\alpha^k_j|$ in Fig. 4, are rather "symmetric," i.e., those of the k-th and (N + 1 − k)-th vibration modes are very similar ($1 \leq k \leq N$), and the order of eigenvectors have actually no effect in the interpretation of the results, and that the "town effect" may induce stronger vibrations only to specific buildings in the town at lower frequencies than expected from conventional analyses.

This study clearly shows that, in analysing dynamic interaction between structures and the deformable ground, not only structural vibrations but also wave propagation in the ground should be taken into account. As seen in Appendix, the complexity of fully-coupled dynamic analyses requires cumbersome calculations that tend to be avoided in many theoretical and numerical studies. However, it should be noted, for instance, that even when we perform some fancy laboratory or numerical experiments using real- or small-scale structures built directly on a shaking table that is subjected to "realistic" and "observed" seismic vibrations, it might be hard to correctly interpret the data obtained by such experiments as long as the shaking table is rigid and the dynamic structure-wave-structure interaction is most likely excluded. In these experiments, we might be able to only obtain some results equivalent to ξ_0 (higher frequencies for a single building) in our present model.

The purpose of the present contribution, however, is not to negate the important results obtained by conventional analyses of engineering seismology. There are certain limitations in the present model: the effect of the amplitude and duration content of the dynamic motion at a site, as well as that of rotation and vertical movement of the foundation of the building is neglected. Furthermore, the foundation itself is assumed to be rigid without any torsion, and no damper (or similar mechanical models) is incorporated. In the further analysis, not only the exact information about each building (e.g., type, fragility, construction year and code, etc.) but also more precise geological and topographical features should be taken into consideration, too. For example, the damage pattern in the Friuli region may be generated also by a standing wave with a nodal or reflection point at the base of the mountain (H.P. Rossmanith, private communication, 2010): All the buildings located at the vibration maxima were destroyed while those located at the nodal points remained more or less intact. Or, there might have been some wave focusing phenomena induced by the local geological conditions. Stratified layers may induce Love waves instead of anti-plane shear waves, and Rayleigh waves with dispersion may become influential near the free surface (Uenishi 2010). However, a simple mass-elastic column system may well explain the collapse of an underground station in Kobe caused by the 1995 Hyogo-ken Nanbu earthquake (Uenishi and Sakurai 2000), and at least the damage distribution in Fig. 1 may be explained in a constructive way with the current simplified model. Hence, it may be concluded that the analysis does indicate the importance of analysing collective behaviour of a town or a structural complex.

4. Conclusions

It has been shown that the collective mechanical behaviour of a group of structures subjected to anti-plane horizontal displacements may be different from the ones expected through conventional seismic analyses. As an example, the generation mechanism of the unique structural damage distribution in the Friuli region observed in 1976 has been studied, and it has been suggested that the structures actually may have shown the dynamic collective behaviour called the "town effect" or "city effect." The model in this study is indeed quite simplified, but nonetheless, it may still hold the fundamental characteristics that will play a crucial role in understanding the seismic performance of a group of structures in urbanised areas. In such areas around the world, the number of skyscrapers, often standing close to each other with comparable heights and mechanical properties, is increasing. The results of the present study may be of use in analysing the dynamic collective performance of such tall buildings on the surface. Also, in the ground, complex structures like dual tunnels are continuously being constructed, and therefore, the "town effect" in rock mass or soil should be investigated.

Appendix: Mathematical Background

In this Appendix, the mathematical treatment of the elastodynamic problem in Chapter 2 is briefly described. Utilising the theory of the single layer potential and taking the effect of the edges of foundations into account, the harmonic solution that satisfies Eq. (1) and the outgoing Sommerfeld radiation condition at infinity may be expressed in a general form as (Ghergu and Ionescu 2009)

$$w(x,y,t) = \mathrm{Re}\left[\frac{i}{4}e^{i\omega t}\sum_{j=1}^{N}\int_{a_j}^{b_j} H_0^{(1)}(\omega\sqrt{(x-s)^2+y^2}/c_S)\frac{\phi(s)}{\sqrt{(s-a_j)(b_j-s)}}ds\right], \qquad (5)$$

where $H_0^{(1)}(x)$ is the Hankel function of the first kind of order zero, and $\phi(x)$ is a continuous function on $a_j < x < b_j$, to be determined by the boundary conditions.

Upon modifications and corrections of the method (and results) presented in (Ghergu and Ionescu 2009), the boundary conditions posed by the rigid foundations of the buildings (2) and the mass-spring-foundation system (3) read

$$\left[\xi^2 - \left(\frac{(c_S)_b}{c_S}\right)^2 \left(\frac{l_b}{h}\right)^2\right] \tau_k(\xi) = 2\xi^2 \frac{\rho_b}{\rho} \frac{h}{l_b} \left\{\xi^2 - \left(1 + \frac{m_0}{m_1}\right)\left[\xi^2 - \left(\frac{(c_S)_b}{c_S}\right)^2 \left(\frac{l_b}{h}\right)^2\right]\right\}, \tag{6}$$

to be satisfied for the k-th vibration mode of the town ($1 \le k \le N$) at $\xi = \xi_k$ (> 0), with $\tau_k(\xi)$ ($\tau_1 \le \tau_2 \le ... \le \tau_N$) being the eigenvalues of the $N \times N$ matrix T that is associated with the definite integral in Eq. (3) and expressed as

$$T_{j,k} = \mathrm{Re}\left[\frac{1}{2}\sum_{p=1}^{2M}\sum_{q=1}^{2M}[\arcsin(\bar{x}_p + 1/(2M)) - \arcsin(\bar{x}_{p-1} + 1/(2M))] M^{-1}_{p+2M(j-1),q+2M(k-1)}\right].$$

$$(1 \le j \le N,\ 1 \le k \le N) \tag{7}$$

Here, $\bar{x}_p = (2p-1)/(2M) - 1$ for $0 \le p \le 2M$, and the $2MN \times 2MN$ matrix $M^{-1}{}_{p+2M(j-1),q+2M(k-1)}$ is the inverse of $M_{p+2M(j-1),q+2M(k-1)}$ that is given by

$$M_{p+2M(j-1),q+2M(k-1)} = -\frac{J_0(\xi|g_j(\bar{x}_p) - g_k(\bar{x}_q)|)}{2\pi\sqrt{1-\bar{x}_q}} \int_{\bar{x}_{q-1}+1/(2M)}^{\bar{x}_q+1/(2M)} \frac{\ln|g_j(\bar{x}_p) - g_k(u)|}{\sqrt{1+u}} du$$

$$+ \left[A_0(\xi|g_j(\bar{x}_p) - g_k(\bar{x}_q)|) - \frac{J_0(\xi|g_j(\bar{x}_p) - g_k(\bar{x}_q)|)}{2\pi} \ln\frac{\xi}{2}\right]$$

$$\cdot [\arcsin(\bar{x}_q + 1/(2M)) - \arcsin(\bar{x}_{q-1} + 1/(2M))], \quad (\text{for } 1 \le q \le M)$$

$$M_{p+2M(j-1),q+2M(k-1)} = -\frac{J_0(\xi|g_j(\bar{x}_p) - g_k(\bar{x}_q)|)}{2\pi\sqrt{1+\bar{x}_q}} \int_{\bar{x}_{q-1}+1/(2M)}^{\bar{x}_q+1/(2M)} \frac{\ln|g_j(\bar{x}_p) - g_k(u)|}{\sqrt{1-u}} du$$

$$+\left[A_0(\xi\left|g_j(\bar{x}_p)-g_k(\bar{x}_q)\right|)-\frac{J_0(\xi\left|g_j(\bar{x}_p)-g_k(\bar{x}_q)\right|)}{2\pi}\ln\frac{\xi}{2}\right]$$

$$\cdot[\arcsin(\bar{x}_q+1/(2M))-\arcsin(\bar{x}_{q-1}+1/(2M))], \qquad (\text{for } M+1\le q\le 2M)$$

(8)

for $1 \le j \le N$, $1 \le k \le N$ and $1 \le p \le 2M$, with $g_j(u) = [a_j + b_j - (a_j - b_j)\, u]/(2l_b)$, $J_0(x)$ being the Bessel function of the first kind of order zero, $A_0(x) = i\, H_0^{(1)}(x) / 4 + J_0(x) \ln(x/2) / (2\pi)$ if $x \neq 0$ and $(i\pi - 2\gamma)/(4\pi)$ if $x = 0$, and finally, γ is the Euler-Mascheroni constant (γ = 0.57721566...). In obtaining Eq. (8), J_0, A_0 and ϕ are approximated as constant functions on each interval $l_b g_j(\bar{x}_{p-1}+1/(2M)) < x < l_b g_j(\bar{x}_p+1/(2M))$ along the rigid foundation of the j-th building ($1 \le j \le N$, $1 \le p \le 2M$). This way of discretisation gives fast convergence of the calculation, and with relatively smaller M precise results may be obtained (Ghergu and Ionescu 2009). In the calculations in this study, M = 100 is used.

From Eq. (6), the normalised eigenfrequency ξ_k [or $f_k = \xi_k c_S/(2\pi l_b)$ in a dimensional form] associated with the k-th vibration mode of the town is obtained through the eigenvalue $\tau_k(\xi_k)$. Note that ξ_k is controlled not only by m_1/m_0, ρ_b/ρ and $(c_S)_b/c_S$ but also implicitly by d/l_b, h/l_b and l_t/l_b (or N) through the function g_j, i.e., $a_j = -l_t + (2l_b + d)\,(j - 1)$ and $l_t = b_N = a_N + 2l_b$. The normalised eigenvectors α^k_j associated with the matrix $T_{j,k}(\xi = \xi_k)$ and its eigenvalue $\tau_k(\xi_k)$ give the normalised displacement amplitudes of the k-th vibration mode of the foundation j

as $w^{m0}{}_j(t) = \alpha^k{}_j\, e^{i\omega t}$ (and $w^{m1}{}_j(t) = \alpha^k{}_j/\{1 - \xi_k{}^2(h/l_b)^2[c_S/(c_S)_b]^2\}\, e^{i\omega t}$ for the mass at the top), and then the function ϕ for that vibration mode is obtained in a discretised form as $\phi(x) = \mathrm{Re}\left[\sum_{l=1}^{N}\sum_{q=1}^{2M} M^{-1}_{p+2M(j-1),q+2M(l-1)}\alpha^k_l\right]$ for $l_b g_j(\bar{x}_{p-1} + 1/(2M)) < x < l_b g_j(\bar{x}_p + 1/(2M))$ (again, $1 \le j \le N$, $1 \le p \le 2M$), and the displacement in the homogeneous, isotropic linear elastic half-space $w(x, y, t)$ may be calculated using Eq. (5) for the k-th vibration mode and normalised eigenvectors $\alpha^k{}_j$.

Equation (6) implies that $\xi = [(c_S)_b / c_S](l_b / h)\sqrt{1 + m_1 / m_0}$ when there is no structure-wave-structure interaction [i.e., $\tau_k(\xi) = 0$ and hence the eigenfrequency for a single m_1-k-m_0 (mass-spring-foundation) system] and $\xi = [(c_S)_b / c_S](l_b / h)$ when $1/\tau_k(\xi) \to 0$ [normalised eigenfrequency for a single (or equivalently an N-) m_1-k (mass-spring) system on top of a rigid half-space where no displacement is allowed and the structure-wave-structure interaction is "infinite"]. That is, the eigenfrequencies ξ_k of the N building system lie in the range

$$\frac{(c_S)_b}{c_S}\frac{l_b}{h}(\equiv \xi_\infty) < \xi_N \le \xi_{N-1} \le \ldots \le \xi_1 < \frac{(c_S)_b}{c_S}\frac{l_b}{h}\sqrt{1 + \frac{m_1}{m_0}}(\equiv \xi_0)\,. \tag{9}$$

It should be noted that the first vibration mode corresponding to the smallest eigenvalue $\tau_1(\xi_1)$ gives the highest eigenfrequency ξ_1 (f_1), and vice versa. In Figs 3 and 4, a lower eigenfrequency is related to more complex "out-of-phase" eigenvectors of the town. The alternate feature mentioned in Discussion ("reverse" or "normal" order of vibration modes) is not recognised in (Ghergu and Ionescu 2009), where the eigenfrequencies are calculated for the cases of $N = 1$, 3 and 21 buildings in a town with the same geometrical and mechanical properties as in this study but the results are inversely presented, i.e., their eigenfrequency for the k-th vibration mode is actually that for the $(N + 1 - k)$-th mode $(1 \leq k \leq N)$.

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Figure Captions

Fig. 1 Structural damage caused by the 1976 Friuli, Italy, earthquake. Astonishingly, each adjacent building has shown fully alternate mechanical behaviour – totally collapsed, almost undamaged, totally collapsed, almost undamaged, ... This structural damage pattern might not be systematically explained by conventional seismic analyses that usually treat each building separately (Photograph courtesy of Prof. H.P. Rossmanith in Vienna).

Fig. 2 The "town model" employed in the analysis (modified after Ghergu and Ionescu 2009): (a) N buildings are uniformly distributed on the flat surface of a linear elastic half-space (rock or soil), with separation distance d. The total length of the town is $2l_t$, and each building j ($1 \leq j \leq N$) is represented by a rigid foundation (having width $2l_b$ but no height), a mass at the top and a linear elastic spring that connects the mass and foundation; and (b) Due to dynamic interaction between the buildings and the anti-plane elastic waves in the ground, each foundation (and mass at the top) may behave mechanically differently even for a single vibration frequency of the town.

Fig. 3 Normalised eigenfrequncy ξ_k ($\equiv 2\pi f_k\, l_b/c_S$) related to the vibration mode k [$1 \leq k \leq N$ (= 7)] of a town that consists of seven identical buildings on a linear elastic half-space [$d/l_b = 0.4$, $h/l_b = 2$, $m_1/m_0 = 1.5$, $\rho_b/\rho = 0.1$ and $(c_S)_b/c_S = 1.5$].

Fig. 4 For the same town with seven buildings, this figure shows the normalised displacement amplitude of the j-th foundation for the vibration mode k [eigenvectors α^k_j (black rectangles) and $|\alpha^k_j|$ (white rectangles where $|\alpha^k_j| = -\alpha^k_j$), with $1 \leq j \leq N$ and $1 \leq k \leq N$ (= 7)]: (a) First; (b) second; (c) third; (d) fourth; (e) fifth; (f) sixth; and the (g) seventh mode.

Fig. 1 (smaller image for the Web version)

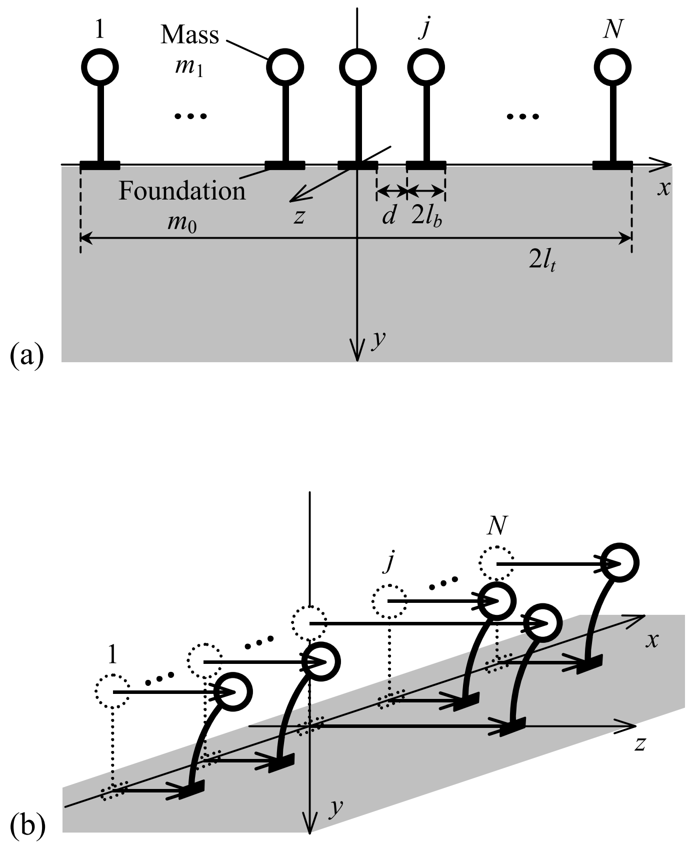
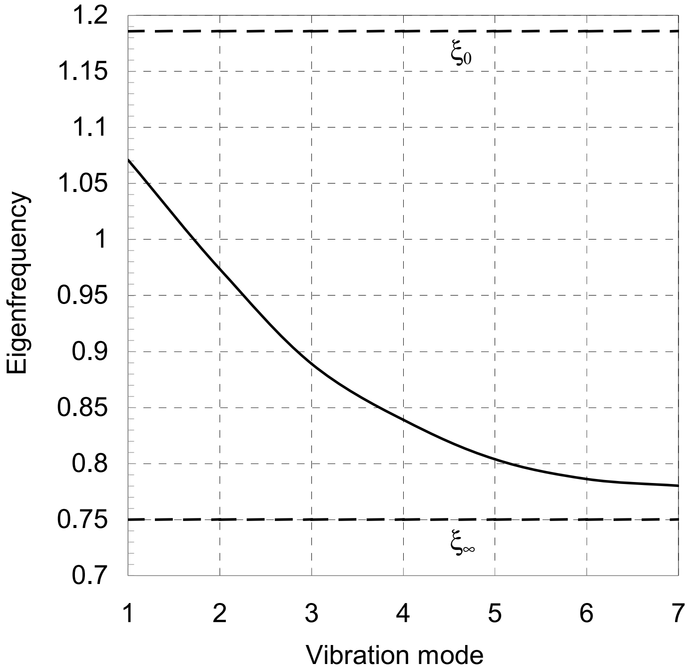


Fig. 2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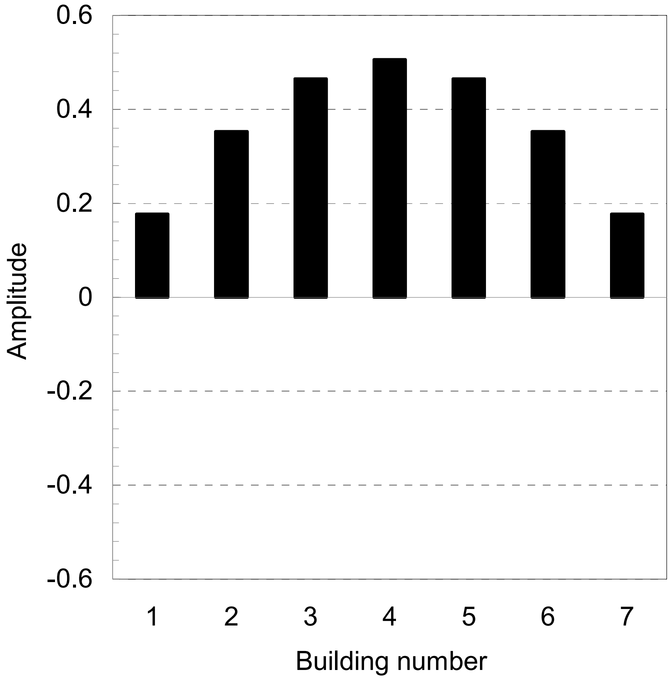
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
Eigenfrequency
ξ0
ξ∞
1
2
3
4
5
6
7
Vibration mode

Fig. 3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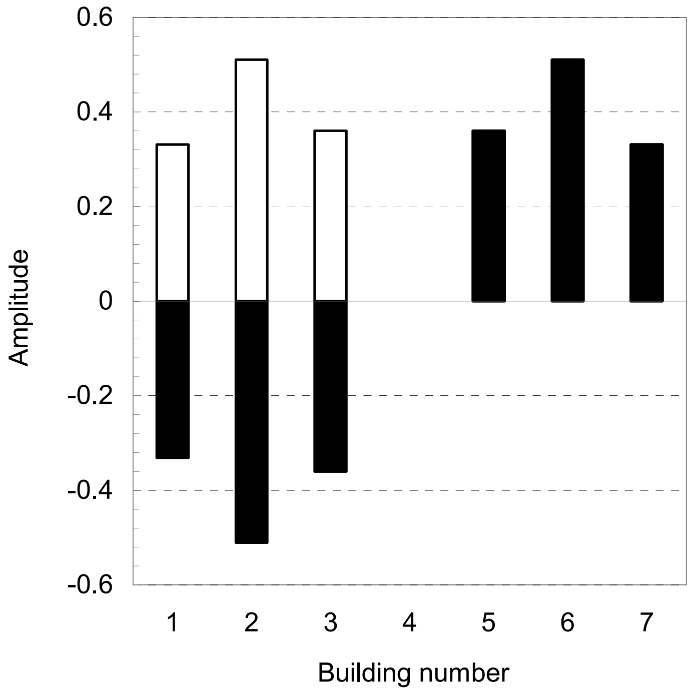
0.4
0.2
0
-0.2
-0.4
-0.6
Amplitude
1
2
3
4
5
6
7
Building number

(a)


0.6
0.4
0.2
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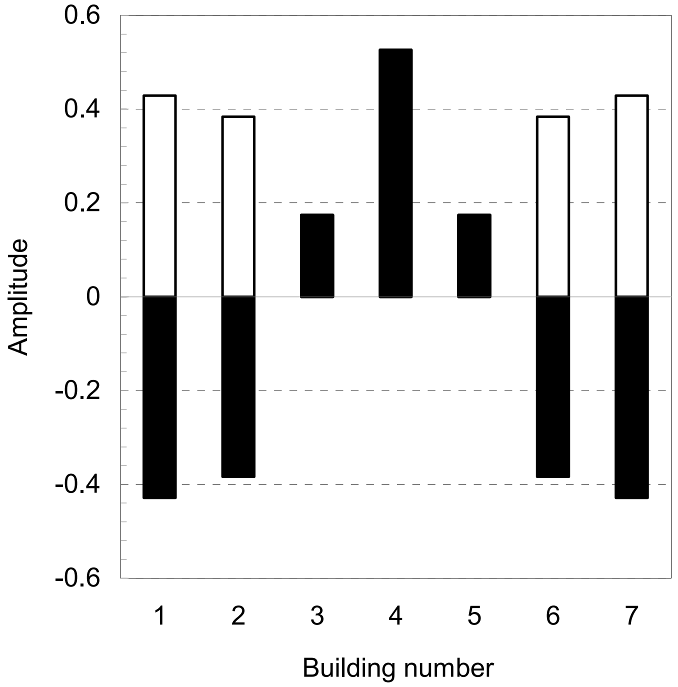
-0.2
-0.4
-0.6
Amplitude
1
2
3
4
5
6
7
Building number

(b)

Fig. 4



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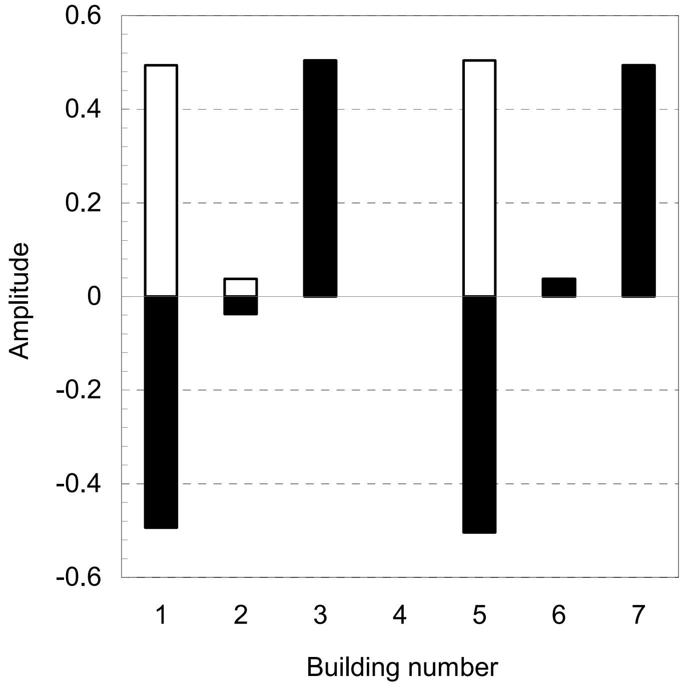
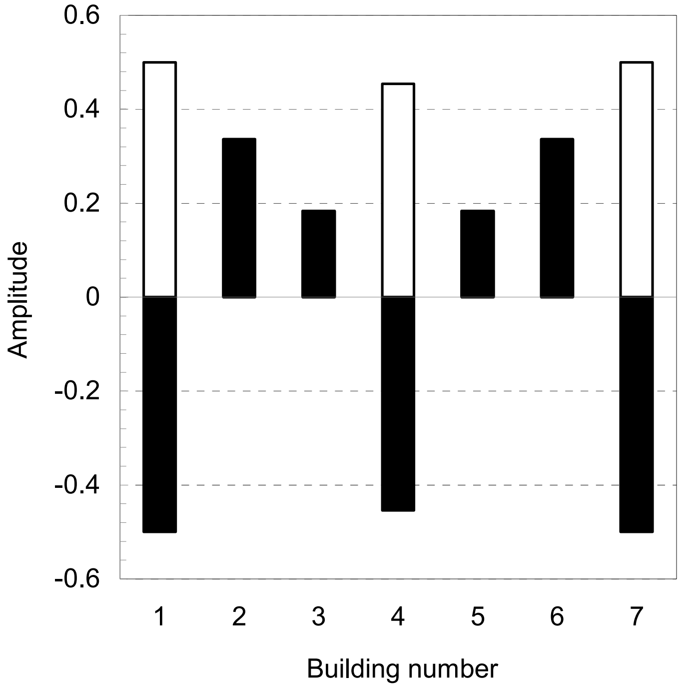


(d)

Fig. 4 (continued)



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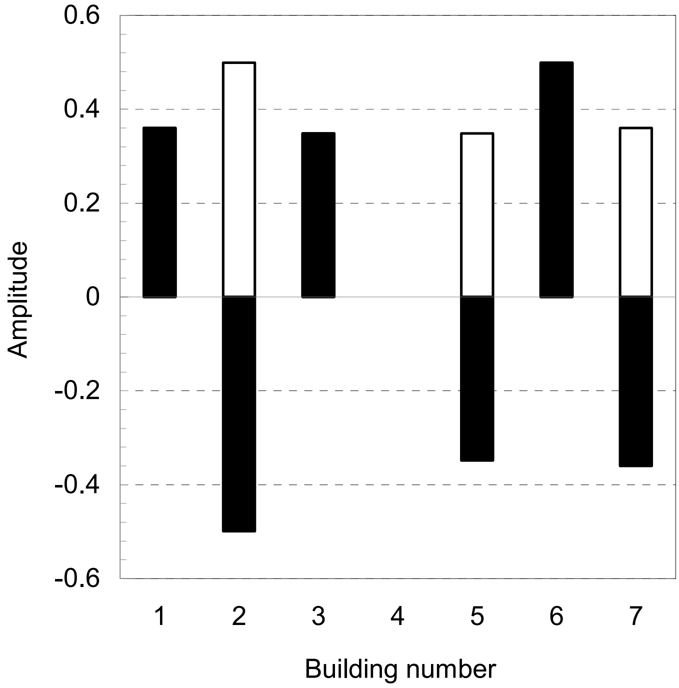
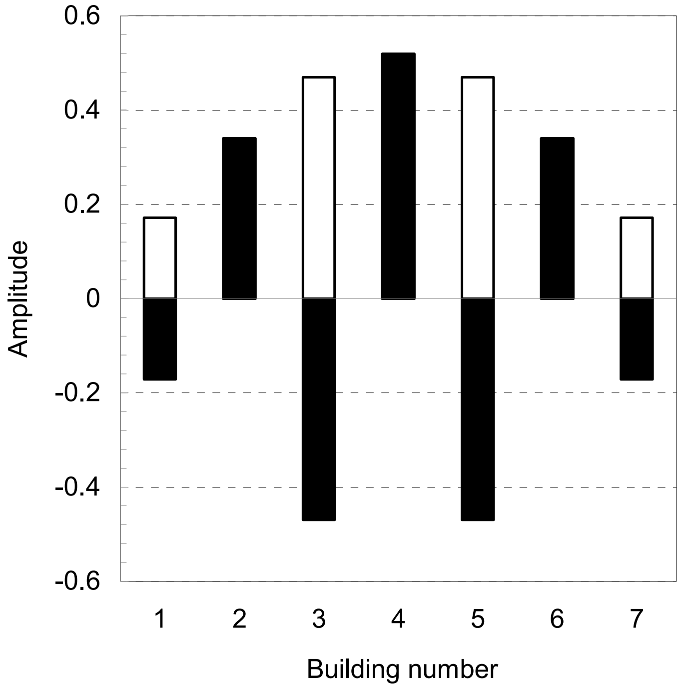


(f)

Fig. 4 (continued)



(g)

Fig. 4 (continued)